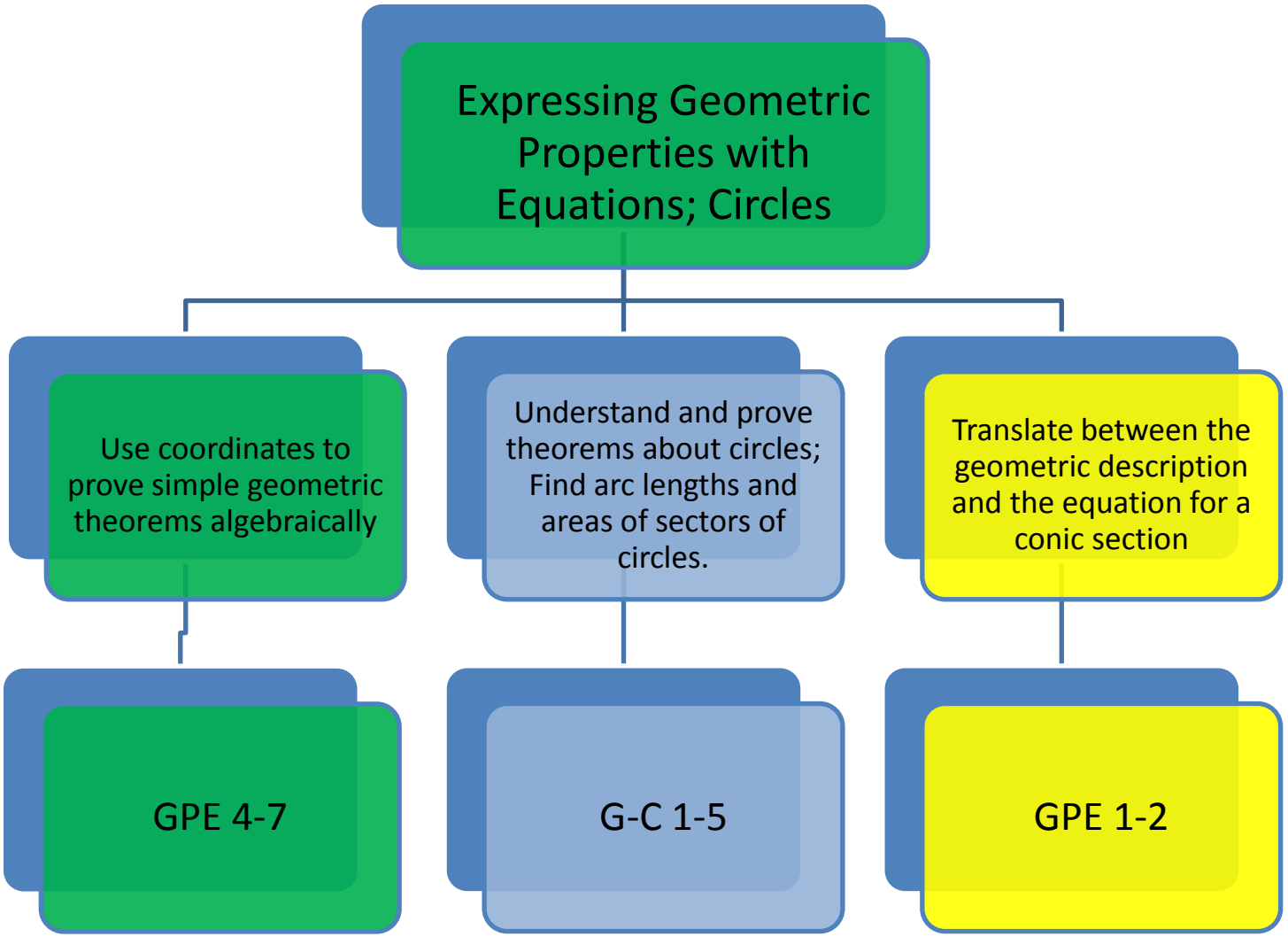
High School Geometry
Unit 3

G-MG 1-3: Modeling with Geometry: Apply geometric concepts in modeling situations



Key: Major Clusters; Supporting Clusters; Additional Clusters

High School Geometry – Unit 3

Express Geometric Properties with Equations; Extend Similarity to Circles

Critical Area: Students investigate triangles and decide when they are similar; with this newfound knowledge and their prior understanding of proportional relationships, they define trigonometric ratios and solve problems using right triangles. They investigate circles and prove theorems about them. Connecting to their prior experience with the coordinate plane, they prove geometric theorems using coordinates and describe shapes with equations. Students extend their knowledge of area and volume formulas to those for circles, cylinders and other rounded shapes. They prove theorems, both with and without the use of coordinates.

CLUSTERS	COMMON CORE STATE STANDARDS
Use coordinates to prove simple geometric theorems algebraically	**Geometry - Expressing Geometric Properties with Equations** **G.GPE.4**. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).* **G.GPE.5**. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). **G.GPE.6**. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **G.GPE.7**. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★
Understand and apply theorems about circles **Find arc lengths and areas of sectors of circles**	**Geometry - Circles** **G.C.1**. Prove that all circles are similar. **G.C.2.** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **G.C.3.** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **G.C.5.** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. **Convert between degrees and radians. CA**
Translate between the geometric description and the equation for a conic section	**Geometry - Expressing Geometric Properties with Equations** **G.GPE.1**. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **G.GPE.2**. Derive the equation of a parabola given a focus and directrix.
MATHEMATICAL PRACTICES	

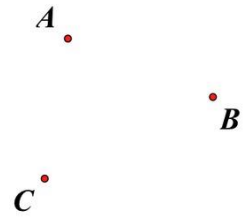
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. **Construct viable arguments and critique the arguments of others.** 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.** 7. **Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	As you begin this unit, it is advised that you start with MP1 and MP 3 to set up your expectations of your classroom. This will help you and your students become proficient in the use of these practices. Emphasize Mathematical Practices 1, 2, 3, 4, 5, 6, and 7 in this unit.

LEARNING PROGRESSIONS

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Express a geometric relationship algebraically (e.g. the Pythagorean Theorem) to new situations such as deriving equation of a circle using the distance formula or deriving the equation of a parabola in terms of focus and directrix. • Right triangle and triangle similarity can be applied to geometric and algebraic theorems to find coordinates of a point on a line given proportion of segments on the line. • Justify algebraically the relationships between slopes of parallel and perpendicular lines as they can be established through proof. • The algebraic representation of a geometric problem can be used to prove theorems in a coordinate plane. • The concept of similarity as it relates to circles can be extended with proof. • Relationships between angles, radii and chords will be investigated. • Similarities will be applied to derive an arc length and a sector area.	• Given coordinate plane information, can we prove (or disprove) geometric relationships (e.g. given the vertices, disprove the assertion that ABCD is a rhombus; or that a given point lies on a circle)? • What is always true about the slopes of perpendicular (or, parallel) lines, and how can a proof be written to exemplify this? • How might we use "constant of proportionality" to define radian measure? • How can we write the equation for a circle or parabola? • How can algebraic representation of a geometric problem be used to prove theorems in coordinate plane? • How can the relationships between angles, radii, and cords be investigated?	• arc • circumscribed • focus • derive • directrix • focus • inscribed • intersect • parallel line • perpendicular line • polygon • Pythagorean Theorem • sector • similar • tangent • vector (directed line segment)

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
LAUSD Adopted Textbooks and Programs • Big Ideas Learning - Houghton Mifflin Harcourt, 2015: Big Ideas Geometry • College Preparatory Mathematics, 2013: Core Connections, Geometry • The College Board, 2014:Springboard Geometry **Materials**: • Compass, straight-edge, graph paper, reflective surface, protractor, tracing paper, scissors, tape. • Geometer's Sketchpad or other software. Geogebra Software **Illustrative Mathematics** Right triangles inscribed in circles II: G.C.2a **http://www.illustrativemathematics.org/illustrati**	Teachers are encouraged to use a variety of strategies for engaging students in understanding and writing proofs, including: using ample pictures or diagrams to demonstrate results and generate strategies; using patty paper, transparencies, or dynamic geometry software to explore the steps in a proof; creating flow charts and other organizational diagrams for outlining a proof; and writing step-by-step or paragraph formats for the completed proof (MP.5). Design an activity where students extend their understanding of the usefulness of similarity transformations through investigating circles (G-C.1). For instance, students can reason that any two circles are similar by describing precisely how to transform one onto the other, as the example	**Formative Assessments** – include checking for understanding using dry-erase boards, exit tickets such as the following activity: You have been asked to place a fire hydrant so that it is an equal distance form three locations indicated on the following map. Show how to fold your paper to physically construct this point as an intersection of two creases (http://www.illustrativemathematics.org/illustrations/508). 

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
ons/1093 Inscribing a triangle in a circle : G.C.**3a** **http://www.illustrativemathematics.org/illustrations/1013** Two Wheels and a Belt : G.C. B **http://www.illustrativemathematics.org/illustrations/621** Equal Area Triangles on the Same Base II : G.GPE.5b **http://www.illustrativemathematics.org/illustrations/1348**	illustrates with two specific circles. **Example.** Students can show that the two circles C and D given by the equations below are similar. $C: (x-1)^2 + (y-4)^2 = 9$ $D: (x+2)^2 + (y-1)^2 = 25$ **Solution.** Since the centers of the circles are $(1, 4)$ and $(-2, 1)$, respectively, we first translate the center of circle C to the center of circle D using the translation $T(x, y) = (x-3, y-3)$. Finally, since the radius of circle C is 3 and the radius of circle D is 5, we dilate from the point $(-2, 1)$ by a scale factor of $5/3$. **Geometry Construction** – Students use a variety of tools and methods to make formal geometric constructions, such as: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	LAUSD Assessment District assessments are under development. More information to come soon. State Assessments California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. There is no assessment for Algebra 1. The 11th grade assessment will include ítems from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit the SMARTER Balance Assessment at: SBAC - http://www.smarterbalanced.org/

LANGUAGE GOALS for **low achieving, high achieving, students with disabilities and English Language Learners**

- Students organize their math thinking and use precise language of mathematics to describe their findings.
- Students communicate their math thinking clearly orally and by writing with their peers and the teacher using academic vocabulary.
- Students will describe in writing the characteristics between inscribed angles and central angles using key vocabulary.
- Students will identify words in word problems that help them formulate arguments; they will use the sentence starter, "The words _____ and _____ lead me to believe…"
- Students will evaluate arguments and make specific claims about slopes of perpendicular and parallel lines.
- Students will compare circles and describe its' similarity using complete sentences and academic language (dilation, ratio).
- Students will make predictions about a problem using predicting verbs and give a reason for their prediction using supporting vocabulary.

PERFORMANCE TASKS

Illustrative Mathematics

Right triangles inscribed in circles I: G.C.2 **http://www.illustrativemathematics.org/illustrations/1091**
Tangent Lines and the Radius of a Circle : G.C.2a, 9 **http://www.illustrativemathematics.org/illustrations/963**
Locating Warehouse : G.C.3a, G.CO.13 **http://www.illustrativemathematics.org/illustrations/507**
Tangent to a circle from a point : G.C.4a **http://www.illustrativemathematics.org/illustrations/1096**
A Midpoint Miracle : G.GPE.4b, 5b **http://www.illustrativemathematics.org/illustrations/605**
Slopes and Circles : G.GPE.1 **http://www.illustrativemathematics.org/illustrations/479**
Explaining the Equation for a Circle : G.GPE.1 **http://www.illustrativemathematics.org/illustrations/1425**
Finding Triangle Coordinates : G.GPE.6, G.SRT.**5** **http://www.illustrativemathematics.org/illustrations/1685**

PERFORMANCE TASKS

LAUSD Concept Lessons

http://math.lausd.net
The Bermuda Triangle
Awesome Amanda

Mathematics Assessment Project (MARS Tasks):

Inscribing and Circumscribing Right Triangles - G.C.3 http://map.mathshell.org/materials/lessons.php?taskid=403&subpage=problem
Geometry Problems: Circles and Triangles-http://map.mathshell.org/materials/lessons.php?taskid=222&subpage=problem
Finding Equations of Parallel and Perpendicular Lines –G.GPE.5 http://map.mathshell.org/materials/lessons.php?taskid=226&subpage=concept
Sectors of Circles-G.C.5 http://map.mathshell.org/materials/lessons.php?taskid=441&subpage=concept
Equations of Circles 1-G.GPE.1 http://map.mathshell.org/materials/lessons.php?taskid=406&subpage=concept

Equations of Circles 2: G.GPE.3.1; MP 1,7: http://map.mathshell.org/materials/lessons.php?taskid=425#task425		
DIFFERENTIATION📖		
UDL/FRONT LOADING	ACCELERATION	INTERVENTION
Assessment tasks can be given a day prior in class or as homework to determine the difficulties students have prior to the lessons. Clarify the objectives in student-friendly language and communicate the learning expectations by the end of the concept development tasks to lower the anxiety. Provide examples of completing the square. Slope, midpoint, distance formulae. Definition of a circle, review circle formulae from prior grade. Review some of the following depending on your students' strength: the coordinate system, solving algebraic equations and inequalities, Pythagorean Theorem, definition of a Parabola, constructing right triangles, triangle similarity, and slopes of parallel and perpendicular lines	Combine trigonometric ratios with the concepts of 30-60-90, 45-45-90 triangles to have students determine the connection between radian and degree measures and developing the Unit Circle. Engage students to apply parabola to focus a signal from either the concave or convex side of the parabola. Determine the value of pi by using the properties of inscribed and circumscribed squares and hexagons. Neglecting the Curvature of the Earth: http://www.illustrativemathematics.org/illustrations/1345	• Multiple entry points for problems should be planned. When the lesson is reviewed or retaught use a different entry point or a different method. • Inquire about students' misconception or misunderstanding before choosing or recommending strategies aligned with math goals and students' abilities. • Use higher order questions and effective questioning techniques to enhance learning, analyzing skills and evaluation. • To increase active participation, students should be expected to work collaboratively to help language learners to lower anxiety, promote authentic conversation, create opportunities for asking questions, and support peers and teachers. • Use visual tools, academic language, graphic organizers, manipulatives, and engaging real world examples to develop interest. • Make clear connections to prior grade concepts of slope, parallel and perpendicular lines, proportional relationship between lengths, what students know about angles, lines. • Explain to students what the markings on diagrams and constructions mean in plain English and in the language of geometry. • Use construction lines to help understand the diagrams. • Name lines, rays, segments. • See Common Issues of each Mars Task

References:

1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics)*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel (Ed.). Retrieved from http://ime.math.arizona.edu/progressions/#committee.
3. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://www.engageny.org/resource/geometry-module-1.
4. Mathematics Assessment Resource Service, University of Nottingham. (2007 - 2012). Mathematics Assessment Project. Retrieved from http://map.mathshell.org/materials/index.php.
5. Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.
6. Partnership for Assessment of Readiness for College and Career. (2012). PARCC Assessments. Retrieved from http://www.parcconline.org/parcc-assessment.
7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.
10. Artzt, Alice F., and Claire M. Newman. How to Use Cooperative Learning in the Mathematics Classroom. Reston, VA: NCTM, 1997.